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Holographic mesons in global Pilch-Warner background geometry

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ABSTRACT: In this paper we study D-brane scalar fluctuations in global Pilch-Warner background geometry. We consider configurations of the probe branes compatible with the kappa symmetry preserving condition and the brane classical equations of motion. The corresponding meson spectra, obtained by the fluctuations along the transverse brane directions, admit equidistant structure for the higher modes, but some of them show additional shifts in their ground states.

KEYWORDS: AdS/CFT correspondence, Gauge/gravity correspondence, D-branes

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1 Introduction

The AdS/CFT correspondence is a fascinating duality relating 10 dimensional IIB string theory in the weak coupling regime to a four dimensional $SU(N)$ gauge field theory with strong coupling constant, and vice-versa. In this case the gauge field theory lives on the boundary of the spacetime where the strings move. This correspondence gives us the opportunity to study non-perturbative phenomena in Yang-Mills theory with tools available in the classical superstring theory and supergravity.

In the original Maldacena setup [1] there is an $\mathcal{N} = 4$ supersymmetric Yang-Mills theory on the gauge side of the correspondence, and a stack of parallel N_c D3-branes on the

string side of the correspondence. Here both endpoints of the strings are attached to the same stack of D3-branes, which allows only adjoint fields. Adding flavours or fundamental matter like quarks can be achieved by introducing a separate stack of N_f D7 probe branes. Such configurations result in appearance of $SU(N_f)$ global flavor symmetry [2]. Now the endpoints of the fundamental strings lie on a different stack of branes. Since they are separated in some directions at finite length, and therefore the strings have finite energy, the quark mass is given by the separation distance times the string tension: $m_q = L/2\pi\alpha'$. If we consider the case where $N_f \ll N_c$ and then take large number of D3-branes, we end up with a strongly coupled dual gauge theory, and a stack of D3-branes sourcing the background geometry. In the limit of large N_c the stack of D3-branes can effectively be replaced by the $AdS_5 \times S^5$ space. This setup allows us to study the N_f D7-branes in the probe limit, where the energy density of the stack of D7-branes does not backreact on the background geometry.

Variety of supersymmetric meson spectra were found in [3, 4] (for an extensive review see [5]). In order to find realistic string theory description of QCD and the Standard model we have to reduce the supersymmetry to some level, but still keeping control on the theory. A way to achieve this goal is to consider deformations of the initial $AdS_5 \times S^5$ geometry [6, 7], or involvement of external magnetic or electric fields [8–11]. Such configurations will break the supersymmetry and theories with less supersymmetry will emerge. In this context Pilch-Warner geometry [12–14] is a fine example of such deformed geometry. It is a solution of five-dimensional $\mathcal{N} = 8$ gauged supergravity lifted to ten dimensions and preserving 1/4 of the original supersymmetry in its infrared critical point.

The rich structure of the theory on both sides of the AdS/CFT correspondence promises interesting results. The recent revival interest to the holographic dual of Pilch-Warner geometry [15–19] also motivates our study.

This paper is organized as follow. First we give some details about Pilch-Warner background. After that in section 3 we find the explicit form of the kappa symmetry matrix for both the D5 and D7 probe branes. Then we solve the kappa symmetry preserving condition. Its solution allows us to find embeddings of the branes compatible with the kappa symmetry and the classical equations of motion. We also prove that the kappa symmetry preserves exactly 1/2 of the spinor degrees of freedom for both D5 and D7.

In section 4 we consider the kappa symmetric D7-brane embedding and study the corresponding spectrum of scalar fluctuations. We show that the ground state of the spectra along the ϕ and θ directions are equidistant for the higher modes. However, an unexpected shift appears in the ground state of the spectrum along the θ direction.

Analogously, in section 5 we study the kappa symmetric D5 embedding and analyze the spectrum of the corresponding fluctuations.

We conclude by briefly discussing the results in section 6.

Finally, there are number of appendices. Appendix A contains the explicit form of the R-R and NS-NS potentials written in global Pilch-Warner coordinates. In Appendices B and C we give the explicit representation of the ten dimensional Dirac gamma matrices and the vielbein coefficients of the PW metric. In Appendices D and E we present in details the form of the kappa symmetry matrices for D7 and D5 probe branes.

2 General Setup

This section is intended to briefly introduce the the Pilch-Warner background geometry, D-branes embeddings and their field content. First we introduce some details about the ten dimensional PW background. Then we introduce D5 and D7 probes. We briefly discuss the form of the non-trivial Ramond-Ramond (R-R) and Neveu-Schwarz (NS-NS) fields entering the D-branes effective actions.

2.1 Pilch-Warner geometry

Pilch-Warner geometry is a solution of five-dimensional $\mathcal{N} = 8$ gauged supergravity lifted to ten dimensions. In the UV critical point it gives the maximally supersymmetric $AdS_5 \times S^5$, while in the infrared point (IR) of the flow it describes warped AdS_5 times squashed S^5 . In this paper we will restrict ourselves to the IR critical point. An important feature on the gravity side is that it preserves $1/8$ supersymmetry everywhere along the flow, while at IR fixed point it is enhanced to $1/4$. On the SYM side the IR point corresponds to the large N limit of the superconformal $\mathcal{N} = 1$ theory of Leigh-Strassler [20].

The warped AdS_5 part of the ten-dimensional PW metric is written by

$$ds_{1,4}^2 = \Omega^2 \left(e^{2A} ds^2(\mathbb{M}^4) + dr^2 \right), \quad (2.1)$$

while the squashed five-sphere part is

$$ds_5^2 = \frac{L_0^2 \Omega^2}{\bar{\rho}^2 \cosh^2 \chi} \left[d\theta^2 + \frac{\bar{\rho}^6 \cos^2 \theta}{X} (\sigma_1^2 + \sigma_2^2) + \frac{\bar{\rho}^{12} \sin^2 2\theta}{4 X^2} \left(\sigma_3 + \frac{2 - \bar{\rho}^6}{2 \bar{\rho}^6} d\phi \right)^2 + \frac{\bar{\rho}^6 \cosh^2 \chi}{16 X^2} (3 - \cos 2\theta)^2 \left(d\phi - \frac{4 \cos^2 \theta}{3 - \cos 2\theta} \sigma_3 \right)^2 \right]. \quad (2.2)$$

Here the function Ω is called the warp factor:

$$\Omega^2 = \frac{X^{1/2} \cosh \chi}{\bar{\rho}}, \quad X(r, \theta) = \cos^2 \theta + \bar{\rho}^6 \sin^2 \theta. \quad (2.3)$$

Our left-invariant one-forms σ_i satisfy $d\sigma_i = \varepsilon_{ijk} \sigma_j \wedge \sigma_k$, so that $d\tilde{\Omega}_3^2 = \sigma_i \sigma_i$ is the metric on the unit 3-sphere. We can choose them as

$$\sigma_1 = \frac{1}{2} (\sin \beta d\alpha - \cos \beta \sin \alpha d\gamma), \quad (2.4a)$$

$$\sigma_2 = -\frac{1}{2} (\cos \beta d\alpha + \sin \beta \sin \alpha d\gamma), \quad (2.4b)$$

$$\sigma_3 = \frac{1}{2} (d\beta + \cos \alpha d\gamma). \quad (2.4c)$$

At the IR point one has $r \rightarrow -\infty$, $\chi = \text{arccosh}(2/\sqrt{3})$, $\bar{\rho} = 2^{1/6}$, and $A(r) = r/L$, thus the metric can be written in the form

$$ds_{1,4}^2 = \Omega^2 \left(e^{\frac{2r}{L}} ds^2(\mathbb{M}^4) + dr^2 \right), \quad (2.5)$$

$$ds_5^2 = \frac{2}{3} L^2 \Omega^2 \left[d\theta^2 + \frac{4 \cos^2 \theta}{3 - \cos 2\theta} (\sigma_1^2 + \sigma_2^2) + \frac{4 \sin^2 2\theta}{(3 - \cos 2\theta)^2} \sigma_3^2 + \frac{2}{3} \left(d\phi - \frac{4 \cos^2 \theta}{3 - \cos 2\theta} \sigma_3 \right)^2 \right]. \quad (2.6)$$

where the AdS radius L is given in terms of the AdS radius L_0 of the UV spacetime by $L = (3/2^{5/3}) L_0$. As shown in [21] there is a natural global $U(1)_\beta$ action $\beta \rightarrow \beta + \text{const}$, which rotates σ_1 into σ_2 , but leaves σ_3 invariant. We adopt the set up where the S^3 Euler angle $\beta \rightarrow \beta + 2\phi$ is shifted to give a solution with a global $U(1)_R = U(1)_\phi$ symmetry. Performing this coordinate transformation on the solution (2.5) and (2.6) we arrive at the final result for the Pilch-Warner metric in global coordinates at the IR fixed point:

$$ds_{1,4}^2(IR) = L^2 \Omega^2 \left(-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_3^2 \right), \quad (2.7)$$

$$ds_5^2(IR) = \frac{2}{3} L^2 \Omega^2 \left[d\theta^2 + \frac{4 \cos^2 \theta}{3 - \cos 2\theta} (\sigma_1^2 + \sigma_2^2) + \frac{4 \sin^2 2\theta}{(3 - \cos 2\theta)^2} (\sigma_3 + d\phi)^2 + \frac{2}{3} \left(\frac{1 - 3 \cos 2\theta}{\cos 2\theta - 3} \right)^2 \left(d\phi - \frac{4 \cos^2 \theta}{1 - 3 \cos 2\theta} \sigma_3 \right)^2 \right], \quad (2.8)$$

where $d\Omega_3^2 = d\phi_1^2 + \sin^2 \phi_1 (d\phi_2^2 + \sin^2 \phi_2 d\phi_3^2)$ is the metric on the unit 3-sphere, and

$$\Omega^2 = \frac{2^{1/3}}{\sqrt{3}} \sqrt{3 - \cos 2\theta} \quad (2.9)$$

represents the warp factor at the IR point.

As a supergravity solution the Pilch-Warner background includes non-trivial Ramond-Ramond (R-R) and Neveu-Schwarz (NS-NS) form fields. The field content of the IIB string theory contains the following R-R potentials: C_0, C_2, C_4, C_6, C_8 with their corresponding field strengths, and the NS-NS Kalb-Ramond two-form B_2 . These fields satisfy certain Bianchi identities and equations of motion listed below: [22]:

$$\Phi = C_0 = 0, \quad F_1 = dC_0 = 0, \quad (2.10a)$$

$$C_2 = \Re e(A_2), \quad B_2 = \Im m(A_2), \quad (2.10b)$$

$$H_3 = dB_2, \quad F_3 = dC_2 - C_0 \wedge H_3 = dC_2, \quad (2.10c)$$

$$dF_3 = dH_3 = 0, \quad dF_5 = H_3 \wedge F_3, \quad (2.10d)$$

$$d(\star F_3) = -H_3 \wedge F_5, \quad d(\star H_3) = F_3 \wedge F_5, \quad F_5 = \star F_5, \quad (2.10e)$$

$$dC_4 + d\tilde{C}_4 = F_5 + C_2 \wedge H_3, \quad (2.10f)$$

$$F_7 = \star F_3 = dC_6 - C_4 \wedge H_3, \quad (2.10g)$$

$$F_9 = \star F_1 = 0 = dC_8 - C_6 \wedge H_3 = C_6 \wedge H_3, \quad \chi = C_8 = 0. \quad (2.10h)$$

Here the field strengths are defined in terms of the corresponding potentials as

$$H_3 \equiv dB_2, \quad F_p \equiv dC_{p-1} - C_{p-3} \wedge H_3. \quad (2.11)$$

In this setup the axion/dilaton system of scalars (2.10a) and (2.10h) is trivial along the flow. We also have an ansatz for the self-dual five form

$$F_5 = -\frac{2^{5/3}}{3} L^4 \cosh \rho \sinh^3 \rho (1 + \star) d\tau \wedge d\rho \wedge \epsilon(S_\phi^3), \quad (2.12)$$

where $\epsilon(S_\phi^3) = \sin^2 \phi_1 \sin \phi_2 d\phi_1 \wedge d\phi_2 \wedge d\phi_3$ is the volume element of the unit 3-sphere S_ϕ^3 , and \star represents the Hodge star operator. The ansatz for the 2-form potential A_2 at the IR point is given by

$$A_2(IR) = C_2 + i B_2 = -\frac{i}{2} e^{-2i\phi} L_0^2 \cos \theta \left(d\theta - \frac{2i \sin 2\theta}{3 - \cos 2\theta} (\sigma_3 + d\phi) \right) \wedge (\sigma_1 + i \sigma_2). \quad (2.13)$$

Two additional constraints are necessary for (2.10a) and (2.10h) to be consistent, namely

$$F_3 \wedge \star H_3 = 0 \quad \text{and} \quad F_3 \wedge \star F_3 = H_3 \wedge \star H_3. \quad (2.14)$$

The explicit form of the RR and the NS-NS potentials in PW satisfying the equations above can be found in Appendix A.

2.2 D-brane embeddings and their field content

As we mentioned in the previous subsection the Pilch-Warner background includes non-trivial Ramond-Ramond (R-R) and Neveu-Schwarz (NS-NS) form fields. Their pullbacks on the D-brane world volume will induce certain terms in the action. The full D-brane effective action, governing the low energy dynamics of the Dp -branes, is given by

$$S_{Dp} = -T_p \int d^{p+1} \xi e^{-\Phi} \sqrt{-\det(\mathcal{G} + \mathcal{F})} + \mu_p \int \sum_n P[C_{(n)}] \wedge e^{\mathcal{F}}, \quad (2.15)$$

where $\mathcal{F} = \mathcal{B}_{(2)} + 2\pi\alpha' F_{(2)}$ is the invariant gauge two-form, $F_{(2)}$ is the world volume gauge field, and $\mathcal{B}_{(2)}$ is the pullback of the Kalb-Ramond field. The relation between the Dp -brane tension T_p ¹ and its charge μ_p is fixed by the supersymmetry: $\mu_p = \pm T_p$. We also have the dilaton Φ and the pullback \mathcal{G} of the background metric:

$$\mathcal{G}_{ab} = G_{AB} \frac{\partial X^A}{\partial \xi^a} \frac{\partial X^B}{\partial \xi^b}. \quad (2.16)$$

The indices $a, b = 0, \dots, p$ span the world volume of the Dp -brane, while $A, B = 0, \dots, 9$ span the whole spacetime. Often it is more useful to work in static gauge, $X^a = \xi^a$, where the pullback is given by

$$\mathcal{G}_{ab} = g_{ab} + G_{mn} \frac{\partial X^m}{\partial \xi^a} \frac{\partial X^n}{\partial \xi^b}. \quad (2.17)$$

¹First calculated in [23].

Here g_{ab} is the induced metric on the D-brane world volume and G_{mn} are the metric components in front of the transverse coordinates governing the D p -brane fluctuations.

The first term in (2.15) is the Dirac-Born-Infeld (DBI) action, and the second term is the Wess-Zumino (WZ) action [22, 23]. The latter has to be expanded in formal series where only $(p+1)$ -forms are selected. For example the WZ part of the action for the D7-brane takes the form

$$S_{WZ} = -T_7 \int \left(P[C_8] - P[C_6] \wedge \mathcal{F} + \frac{1}{2} P[C_4] \wedge \mathcal{F} \wedge \mathcal{F} + \dots \right), \quad (2.18)$$

where P denotes the pullback of the bulk spacetime tensor to the world-volume of the brane. In general one has

$$P[C_{p+1}]_{a_1 \dots a_{p+1}} = \frac{1}{(p+1)!} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{A_1} \dots \partial_{a_{p+1}} X^{A_{p+1}} C_{A_1 \dots A_{p+1}}(X), \quad (2.19)$$

where $\varepsilon^{a_1 \dots a_{p+1}}$ is the antisymmetric Levi-Chevita symbol.

In what follows we would like our D7 probe to extend on the whole warped AdS_5 part of the space, while wrapping a three sphere of the squashed S^5 spanned by the angles (α, β, γ) . This leave us with the following general D7-brane embedding in global Pilch-Warner coordinates:

$$\xi^a = (\tau, \rho, \phi_1, \phi_2, \phi_3, \alpha, \beta, \gamma), \quad \theta = \theta(\xi^a), \quad \phi = \phi(\xi^a), \quad (2.20)$$

where ξ^a are the world volume coordinates, and (θ, ϕ) are the normal coordinates of the brane.

The most general embedding of the D5-brane, that we are going to consider, is given by

$$\xi^a = (\tau, \rho, \phi_2, \phi_3, \beta, \gamma), \quad \phi_1 = \phi_1(\xi^a), \quad \theta = \theta(\xi^a), \quad \alpha = \alpha(\xi^a), \quad \phi = \phi(\xi^a), \quad (2.21)$$

where ξ^a , $a = 0, \dots, 6$, are the D5 world volume coordinates. In global Pilch-Warner this corresponds to a D5-brane embedding wrapping warped AdS_4 subspace and a two-sphere of the squashed S^5 .

3 Kappa symmetry matrix

In this section we consider the implications and the importance of the local fermionic kappa symmetry. Our goal is to find kappa symmetric embeddings of the D5 and D7 probe branes by solving the kappa symmetry preserving condition.

Our D-brane effective action has an additional local fermionic gauge symmetry – kappa symmetry. It implies that half of the components of the background killing spinor ϵ are actually gauge degrees of freedom and can be gauged away. This symmetry is required for the preservation of spacetime covariance and supersymmetry. Kappa symmetry invariance is achieved whenever the background is an on-shell supergravity background, which is the case of the PW geometry. The general formalism to study supersymmetric bosonic world

volume solitons requires that any such configuration must satisfy the kappa symmetry preserving condition [24]

$$\Gamma_k \varepsilon = \varepsilon, \quad (3.1)$$

where $\Gamma_\kappa^2 = 1$ and ε is a Weyl spinor². It is important to stress that kappa symmetry invariance is necessary to define a supersymmetric field theory on the brane, but not sufficient. To determine the supersymmetric configurations of the probe brane the kappa projection condition (3.1) must be compatible with some additional projection conditions for the supergravity background Killing spinor.

Following [25] we have the following expression for the $(p+1)$ -form kappa symmetry matrix:

$$(\Gamma_\kappa)_{(p+1)} = \frac{1}{\sqrt{-\det(\mathcal{G} + \mathcal{F})}} \sum_{\ell=0}^{k+1} \gamma_{(2\ell)} \sigma_3^\ell \wedge e^{\mathcal{F}} i \sigma_2, \quad p = 2k+1, \quad (3.2)$$

where

$$\mathcal{F} = 2\pi\alpha' F_{(2)} + \mathcal{B}_{(2)}, \quad \mathcal{B}_{(2)} = P[B_{(2)}], \quad \mathcal{G} = P[G]. \quad (3.3)$$

The notation $\gamma_{(\ell)}$ stands for the wedge product of the following one-forms:

$$\gamma_{(1)} \equiv d\xi^a \gamma_a = d\xi^a \partial_a X^M E_M^I(X) \Gamma_I. \quad (3.4)$$

We can expand $e^{\mathcal{F}}$ in a formal series of the form

$$e^{\mathcal{F}} = 1 + \mathcal{F} + \frac{1}{2!} \mathcal{F} \wedge \mathcal{F} + \frac{1}{3!} \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} + \frac{1}{4!} \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} + \dots \quad (3.5)$$

Therefore the kappa matrix takes the form

$$(\Gamma_\kappa)_{(p+1)} = \frac{1}{\sqrt{-\det(\mathcal{G} + \mathcal{F})}} \sum_{\ell=0}^{k+1} \gamma_{(2\ell)} \sigma_3^\ell \wedge \left(1 - \mathcal{F} + \frac{1}{2!} \mathcal{F}^2 - \frac{1}{3!} \mathcal{F}^3 + \frac{1}{4!} \mathcal{F}^4 + \dots \right) i \sigma_2, \quad (3.6)$$

where $\mathcal{F}^2 = \mathcal{F} \wedge \mathcal{F}$.

The calculation of the Killing spinor in global PW geometry and the analysis of the supersymmetric brane embeddings will be left for a future work. In what follows we consider only kappa symmetric embeddings compatible with the D5- and D7-brane equations of motion.

3.1 Kappa symmetry for the D7-brane

For the D7-brane one has $p = 8, k = 3$, and only 8-form terms are selected. Thus (3.6) takes the form

$$(\Gamma_\kappa)_{(8)} = \frac{1}{\sqrt{-\det(\mathcal{G} + \mathcal{F})}} \left(\gamma_{(8)} \sigma_3^4 + \gamma_{(6)} \sigma_3^3 \wedge \mathcal{F} + \frac{\gamma_{(4)} \sigma_3^2}{2!} \wedge \mathcal{F}^2 + \frac{\gamma_{(2)} \sigma_3^1}{3!} \wedge \mathcal{F}^3 + \dots \right) i \sigma_2. \quad (3.7)$$

²If we require supersymmetry invariance the spinor ε becomes the background Killing spinor ϵ .

In order to explicitly calculate the D7 kappa symmetry matrix we begin with the most general D7 embedding in terms of global Pilch-Warner coordinates:

$$\xi^a = (\tau, \rho, \phi_1, \phi_2, \phi_3, \alpha, \beta, \gamma), \quad \theta = \theta(\xi^a), \quad \phi = \phi(\xi^a). \quad (3.8)$$

The analysis will greatly simplify if we set the world volume gauge field $F = 0$. The wedge products of the $B_{(2)}$ -form pullback terminates at third order $\mathcal{B}_{(2)} \wedge \mathcal{B}_{(2)} \wedge \mathcal{B}_{(2)} = 0$, and kappa symmetry 8-form matrix (3.7) becomes

$$(\Gamma_\kappa)_{(8)} = \frac{1}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} \left(\gamma_{(8)} \sigma_3^4 + \gamma_{(6)} \wedge \mathcal{B}_{(2)} \sigma_3^3 + \frac{1}{2!} \gamma_{(4)} \wedge \mathcal{B}_{(2)} \wedge \mathcal{B}_{(2)} \sigma_3^2 \right) i \sigma_2. \quad (3.9)$$

After taking all the wedge products one finds

$$(\Gamma_\kappa)_{(8)} = \frac{(M_8 \sigma_3^4 + M_6 \sigma_3^3 + M_4 \sigma_3^2) i \sigma_2}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} d\tau \wedge d\rho \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\alpha \wedge d\beta \wedge d\gamma. \quad (3.10)$$

The Pauli matrices act on the two component Weyl spinor $\varepsilon = \begin{pmatrix} \varepsilon_1^{(\alpha)} \\ \varepsilon_2^{(\beta)} \end{pmatrix}$, $\alpha, \beta = 1, \dots, 16$.

The form of the matrices M_4 , M_6 and M_8 is given in Appendix D. After applying the Pauli matrices the kappa symmetry preserving condition (3.1) takes the form

$$\frac{1}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} \left(M_8 \begin{pmatrix} \varepsilon_2^\beta \\ -\varepsilon_1^\alpha \end{pmatrix} + M_6 \begin{pmatrix} \varepsilon_2^\beta \\ \varepsilon_1^\alpha \end{pmatrix} + M_4 \begin{pmatrix} \varepsilon_2^\beta \\ -\varepsilon_1^\alpha \end{pmatrix} \right) = \begin{pmatrix} \varepsilon_1^\alpha \\ \varepsilon_2^\beta \end{pmatrix}. \quad (3.11)$$

This is an algebraic system for the components of the spinor. We can solve it if we choose a simpler D7-brane embedding ansatz, namely

$$\theta = \theta(\rho), \quad \phi = \phi(\beta). \quad (3.12)$$

In this case the system (3.11) takes the form

$$\sum_{\beta=1}^{16} a_{\alpha\beta} \varepsilon_2^{(\beta)} = \varepsilon_1^{(\alpha)}, \quad \alpha = 1, \dots, 16, \quad (3.13)$$

$$\sum_{\beta=1}^{16} b_{\alpha\beta} \varepsilon_1^{(\beta)} = \varepsilon_2^{(\alpha)}, \quad \alpha = 1, \dots, 16, \quad (3.14)$$

where the coefficients $a_{\alpha\beta}$ and $b_{\alpha\beta}$ are functions of the coordinates. We can substitute the $\varepsilon_2^{(\alpha)}$ from (3.14) into (3.13), so that we end up with a homogenous system of 16 equations for the components $\varepsilon_1^{(\alpha)}$:

$$\sum_{\beta=1}^{16} s_{\alpha\beta} \varepsilon_1^{(\beta)} = 0, \quad \alpha = 1, \dots, 16. \quad (3.15)$$

A homogenous system has a non-trivial solution if the determinant of its matrix is zero. In our case we have to impose $\det(s_{\alpha\beta}) = 0$. This condition implies

$$\theta = 0, \quad \phi = -\beta + c, \quad (3.16)$$

and the solutions are only the constant spinors as shown in Appendix D.

3.2 Kappa symmetry matrix for the D5-brane

In order to find the D5 kappa symmetry matrix we consider the most general D5 embedding ansatz of the form

$$\xi^a = (\tau, \rho, \phi_2, \phi_3, \beta, \gamma), \quad \phi_1 = \phi_1(\xi^a), \quad \theta = \theta(\xi^a), \quad \alpha = \alpha(\xi^a), \quad \phi = \phi(\xi^a). \quad (3.17)$$

The 6-form kappa symmetry matrix for the D5-brane, $p = 5$, $k = 2$, looks like

$$(\Gamma_\kappa)_{(6)} = \frac{1}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} \left(\gamma_{(4)} \wedge \mathcal{B}_{(2)} \sigma_3^2 + \frac{1}{2!} \gamma_{(2)} \wedge \mathcal{B}_{(2)} \wedge \mathcal{B}_{(2)} \sigma_3^1 \right) i \sigma_2. \quad (3.18)$$

After taking the wedge products one finds

$$(\Gamma_\kappa)_{(6)} = \frac{M_2 i \sigma_3^1 \sigma_2 + M_4 i \sigma_3^2 \sigma_2}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} d\tau \wedge d\rho \wedge d\phi_2 \wedge d\phi_3 \wedge d\beta \wedge d\gamma. \quad (3.19)$$

The explicit form of the matrices M_2 and M_4 is given in Appendix E. Applying the Pauli matrices, the projection equation (3.1) becomes

$$M_2 \begin{pmatrix} \varepsilon_2^\beta \\ \varepsilon_1^\alpha \end{pmatrix} + M_4 \begin{pmatrix} \varepsilon_2^\beta \\ -\varepsilon_1^\alpha \end{pmatrix} = \sqrt{-\det(\mathcal{G} + \mathcal{B})} \begin{pmatrix} \varepsilon_1^\alpha \\ \varepsilon_2^\beta \end{pmatrix}. \quad (3.20)$$

Considering the following simpler D5 embedding ansatz

$$\phi_1 = \frac{\pi}{2}, \quad \theta = \theta(\rho), \quad \alpha = \alpha(\phi_2), \quad \phi = \phi(\beta), \quad (3.21)$$

one can show that the system (3.20) has constant solutions if

$$\phi_1 = \frac{\pi}{2}, \quad \theta = 0, \quad \alpha = \frac{\pi}{2}, \quad \phi = -\beta + c. \quad (3.22)$$

This is also compatible with the D5-brane equations of motion.

4 D7-brane scalar fluctuations and the meson spectrum

In this section we consider the kappa symmetric D7-brane embedding and study the corresponding energy spectrum of the scalar fluctuations. We show that the ground state of the spectrum along the θ direction is not equal to the conformal dimension of the operators dual to the fluctuations.

4.1 Fluctuations along ϕ

The D7 embedding ansatz

$$\theta(\rho) = \text{const}, \quad \phi(\beta) = -\beta + c \quad (4.1)$$

satisfies both the kappa symmetry preserving condition and the classical D7-brane embedding equations. Next we are going to redefine the coordinate $\phi \rightarrow \phi + \beta$ in order to consider fluctuations around $\phi = \text{const}$. The fluctuation ansatz is given by

$$\theta = 0 + \eta \Theta, \quad \phi = 0 + \eta \Phi. \quad (4.2)$$

Taking into account the proper shift in the metric we can expand the DBI and the WZ Lagrangians up to quadratic order in the fluctuations and keeping only terms of order $\eta^2 = (2\pi\alpha')^2$ we find³

$$\mathcal{L}_{DBI}^{(2)} = -\mu_7 \frac{\eta^2}{2} \sqrt{-\det g} (G_{\phi\phi} g^{ac} \partial_a \Phi \partial_c \Phi + G_{\theta\theta} g^{ac} \partial_a \Theta \partial_c \Theta), \quad (4.3)$$

where $g = \det(g_{ab})$ is the determinant of the induced metric, $G_{\phi\phi}$ and $G_{\theta\theta}$ are the metric components in front of $d\phi^2$ and $d\theta^2$. The WZ part takes the form

$$\mathcal{L}_{WZ}^{(2)} = h_0 (h_1 \Theta \partial_\beta \Phi + h_2 \Theta \partial_\rho \Theta + h_3 \partial_\beta \Phi \partial_\rho \Theta + h_4 \Theta^2), \quad (4.4)$$

where

$$h_0 = \frac{2^{1/3} L^8 \eta^2 \mu_7 \sin \alpha \sin^2 \phi_1 \sin \phi_2 \sinh^4 \rho}{243 (\cos 2\theta - 3)^3},$$

$$h_1 = 4 \cos^2 \theta (\cos 2\theta - 3) (237 - 377 \cos 2\theta + 55 \cos 4\theta - 3 \cos 6\theta) \theta'(\rho),$$

$$h_2 = -4 \cos^2 \theta (\cos 2\theta - 3) (-134 - 237 \phi'(\beta) + \cos 6\theta (2 + 3 \phi'(\beta)) - \cos 4\theta (34 + 55 \phi'(\beta)) + \cos 2\theta (214 + 377 \phi'(\beta))),$$

$$h_3 = 4 \cos^3 \theta (\cos 2\theta - 3)^2 (53 \sin \theta - 3 \sin 3\theta),$$

$$h_4 = \theta'(\rho) \sin 2\theta (304 + 512 \phi'(\beta) + 2 \cos 8\theta (2 + 3 \phi'(\beta)) - \cos 6\theta (78 + 121 \phi'(\beta)) + \cos 4\theta (556 + 906 \phi'(\beta)) - 3 \cos 2\theta (710 + 1277 \phi'(\beta))).$$

The fluctuations for the scalar field $\Phi(\xi^a)$ are governed by the equation

$$\nabla^a \nabla_a \Phi + (\partial_a G_{\phi\phi}) g^{ab} \partial_b \Phi = 0, \quad (4.5)$$

which is a Klein-Gordon-like equation, where

$$\nabla^a \nabla_a \Phi = \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} g^{ab} \partial_b \Phi). \quad (4.6)$$

After expanding the covariant derivatives in eq. (4.5) one finds

$$-\partial_\tau^2 \Phi + \cosh^2 \rho \tilde{\Delta}_\rho \Phi + \coth^2 \rho \Delta_{\phi_i} \Phi + 3 \cosh^2 \rho \tilde{\Delta}_{\alpha_i} \Phi = 0, \quad (4.7)$$

³For the DBI Lagrangian expansion we used the formula

$$\sqrt{\det(\hat{1} + \hat{M})} = 1 + \frac{1}{2} \text{Tr} \hat{M} - \frac{1}{4} \text{Tr} \hat{M}^2 + \frac{1}{8} \text{Tr}^2 \hat{M} - \frac{1}{8} \text{Tr} \hat{M}^2 \text{Tr} \hat{M} + \frac{1}{32} \text{Tr}^2 \hat{M}^2 + \dots$$

It is derived from the following general formula

$$\sqrt{\det(\hat{1} + \hat{M})} = e^{\frac{1}{2} \ln \det(\hat{1} + \hat{M})} = e^{\frac{1}{2} \text{Tr} \ln(\hat{1} + \hat{M})} = e^{\sum_{j=1}^{\infty} \frac{(-1)^{j+1} \text{Tr}(\hat{M}^j)}{2j}} = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\sum_{j=1}^{\infty} \frac{(-1)^{j+1} \text{Tr}(\hat{M}^j)}{2j} \right)^k.$$

where $\phi_i = (\phi_1, \phi_2, \phi_3)$, $\alpha_i = (\alpha, \beta, \gamma)$, and

$$\tilde{\Delta}_\rho \Phi = \partial_\rho^2 \Phi + (3 \coth \rho + \tanh \rho) \partial_\rho \Phi, \quad (4.8)$$

$$\Delta_{\phi_i} \Phi = \frac{\partial_{\phi_1} (\sin^2 \phi_1 \partial_{\phi_1} \Phi)}{\sin^2 \phi_1} + \frac{\partial_{\phi_2} (\sin \phi_2 \partial_{\phi_2} \Phi)}{\sin^2 \phi_1 \sin \phi_2} + \frac{\partial_{\phi_3}^2 \Phi}{\sin^2 \phi_1 \sin^2 \phi_2}, \quad (4.9)$$

$$\tilde{\Delta}_{\alpha_i} \Phi = \frac{1}{\sin \alpha} \partial_\alpha (\sin \alpha \partial_\alpha \Phi) + \frac{1}{\sin^2 \alpha} \left(\frac{\cos 2\alpha + 7}{8} \partial_\beta^2 \Phi + \partial_\gamma^2 \Phi - 2 \cos \alpha \partial_{\beta\gamma}^2 \Phi \right). \quad (4.10)$$

Separation of variables of the form $\Phi = e^{i\omega\tau} R(\rho) \mathcal{Y}^\ell(S_{\phi_i}^3) Z(\tilde{S}_{\alpha_i}^3)$ leads to the following set of spectral equations:

$$\ddot{T}(\tau) = -\omega^2 T(\tau), \quad (4.11)$$

$$\tilde{\Delta}_{\alpha_i} Z(\alpha_i) = -\nu Z(\alpha_i), \quad (4.12)$$

$$\Delta_{\phi_i} \mathcal{Y}^\ell(\phi_i) = -\ell(\ell+2) \mathcal{Y}^\ell(\phi_i), \quad (4.13)$$

$$R''(\rho) + (3 \coth \rho + \tanh \rho) R'(\rho) + \left(\frac{\omega^2}{\cosh^2 \rho} - \frac{\ell(\ell+2)}{\sinh^2 \rho} - 3\nu \right) R(\rho) = 0, \quad (4.14)$$

where ω is the energy of the fluctuations, ν is the eigenvalue of the operator (4.12), and $\mathcal{Y}^\ell(\phi_i)$ are the hyperspherical harmonics, $\ell \in \mathbb{N}_0$. In order to facilitate the calculation of the conformal dimension Δ and the spectrum, we make the following change of variables $\sinh \rho = r$ in the radial equation (4.14):

$$R''(r) + \frac{3+5r^2}{r(r^2+1)} R'(r) + \left(\frac{\omega^2}{(r^2+1)^2} - \frac{\ell(\ell+2)}{r^2(r^2+1)} - \frac{3\nu}{r^2+1} \right) R(r) = 0. \quad (4.15)$$

There are two independent solutions given in terms of the standard Gaussian hypergeometric function:

$$R(r) = R_+(r) + R_-(r), \quad (4.16)$$

$$R_+(r) = c_1 r^{-2-\ell} (r^2+1)^{-\frac{\omega}{2}} {}_2F_1(a, b; c; z), \quad (4.17)$$

$$R_-(r) = c_2 r^\ell (r^2+1)^{-\frac{\omega}{2}} {}_2F_1(a-c+1, b-c+1; 2-c; z), \quad (4.18)$$

with

$$a = \frac{1}{2} (-\ell - \omega - \sqrt{3\nu+4}), \quad b = \frac{1}{2} (-\ell - \omega + \sqrt{3\nu+4}), \quad c = -\ell, \quad z = -r^2.$$

The only regular solution at the origin $r = 0$ is $R_-(r)$, which makes it our choice for normalizable solution. To assure normalizability at infinity one has to terminate the series of the hypergeometric function at some finite non-negative integer power n . As it is well known the hypergeometric function becomes a polynomial of degree n if one of its first two arguments is a negative integer $-n$, $n \geq 0$. Therefore setting $b-c+1 = -n$ gives the quantization condition and the form of the scalar meson spectrum⁴:

$$\omega = \sqrt{4+3\nu} + 2 + \ell + 2n. \quad (4.19)$$

⁴There exists a symmetry between interchanging the first two arguments of the hypergeometric function, i.e. ${}_2F_1(a, b; c; z) = {}_2F_1(b, a; c; z)$, which means we have the freedom to choose which argument to quantize. In our case we chose to have positive energy without any loss of generality.

By the standard AdS/CFT dictionary one can calculate the conformal dimension of the operators corresponding to Φ from the analysis of the radial equation (4.15) at the boundary $r \rightarrow \infty$. The large r behaviour is determined by the following asymptotic equation

$$R''(r) + \frac{5}{r} R'(r) - \frac{3\nu}{r^2} R(r) = 0, \quad (4.20)$$

with a general solution:

$$R(r) = c_1 r^{-\sqrt{3\nu+4}-2} + c_2 r^{\sqrt{3\nu+4}-2}. \quad (4.21)$$

This solution contains normalizable and non-normalizable parts that behaves as $r^{k_1} = r^{\Delta-4+p}$ and $r^{k_2} = r^{-\Delta+p}$, for some constant p . According to the AdS/CFT dictionary taking the difference of the powers one finds the conformal dimension

$$\Delta = \frac{k_1 - k_2}{2} + 2 = 2 + \sqrt{3\nu+4}, \quad (4.22)$$

where $k_1 = -2 + \sqrt{3\nu+4}$, and $k_2 = -2 - \sqrt{3\nu+4}$. Equation (4.22) allows us to express the spectrum in terms of the conformal dimension:

$$\omega = \Delta + \ell + 2n. \quad (4.23)$$

We can conclude that the energy of the ground state is given by the conformal dimension of the operator dual to the fluctuations. This is what we expect for supersymmetric embeddings of the D7-brane, which is consistent with similar results for the meson spectra found in different background geometries [3, 7]. For higher modes the spectrum is equidistant.

4.2 Fluctuations along θ

Now we proceed with the study of the D7-brane scalar fluctuations along the θ direction. The equation governing the fluctuations is given by

$$-\partial_\tau^2 \Theta + \cosh^2 \rho \tilde{\Delta}_\rho \Theta + \coth^2 \rho \Delta_{\phi_i} \Theta + 3 \cosh^2 \rho \tilde{\Delta}_{\alpha_i} \Theta + 3 \cosh^2 \rho \Theta = 0, \quad (4.24)$$

where

$$\tilde{\Delta}_\rho \Theta = \partial_\rho^2 \Theta + \left(3 \coth \rho + \frac{5}{2} \tanh \rho \right) \partial_\rho \Theta, \quad (4.25)$$

$$\Delta_{\phi_i} \Phi = \frac{\partial_{\phi_1} (\sin^2 \phi_1 \partial_{\phi_1} \Phi)}{\sin^2 \phi_1} + \frac{\partial_{\phi_2} (\sin \phi_2 \partial_{\phi_2} \Phi)}{\sin^2 \phi_1 \sin \phi_2} + \frac{\partial_{\phi_3}^2 \Phi}{\sin^2 \phi_1 \sin^2 \phi_2}, \quad (4.26)$$

$$\tilde{\Delta}_{\alpha_i} \Theta = \frac{1}{\sin \alpha} \partial_\alpha (\sin \alpha \partial_\alpha \Theta) + \frac{1}{\sin^2 \alpha} \left(\frac{\cos 2\alpha + 7}{32} \partial_\beta^2 \Theta + \partial_\gamma^2 \Theta - \partial_{\beta\gamma} \Theta \cos \alpha \right). \quad (4.27)$$

The form of eq. (4.24) assumes separation of variables $\Theta(\xi^a) = e^{i\omega\tau} R(\rho) \mathcal{Y}^\ell(S_{\phi_i}^3) Z(\tilde{S}_{\alpha_i}^3)$. This leads to the following radial equation:

$$R''(\rho) + \left(\frac{5}{2} \tanh \rho + 3 \coth \rho \right) R'(\rho) + \left(\frac{\omega^2}{\cosh^2 \rho} - \frac{\ell(\ell+2)}{\sinh^2 \rho} - 3\nu + 3 \right) R(\rho) = 0. \quad (4.28)$$

where ν is the eigenvalue quantum number for the operator defined in (4.27). Changing the radial variable to $r = \sinh \rho$ we arrive at

$$R''(r) + \frac{6 + 13r^2}{2(r + r^3)} R'(r) + \left(\frac{\omega^2}{(1 + r^2)^2} - \frac{\ell(\ell + 2)}{r^2(1 + r^2)} - \frac{3\nu - 3}{1 + r^2} \right) R(r) = 0. \quad (4.29)$$

Its regular solution is given by

$$R(r) = (r^2 + 1)^{-\frac{1}{8}(3 + \sqrt{16\omega^2 + 9})} r^\ell F(a, b, \ell + 2, -r^2), \quad (4.30)$$

where

$$a = \frac{1}{8} \left(\sqrt{73 + 48\nu} - \sqrt{16\omega^2 + 9} \right) + \frac{\ell}{2} + 1, \quad b = -\frac{1}{8} \left(\sqrt{16\omega^2 + 9} + \sqrt{73 + 48\nu} \right) + \frac{\ell}{2} + 1. \quad (4.31)$$

Normalizability of the solution implies restrictions from which one finds the meson spectrum along θ :

$$\omega^2 = (\Delta + \ell + 2n)^2 - \frac{9}{16}. \quad (4.32)$$

The ground state is determined by

$$\omega_0^2 = \Delta^2 - \frac{9}{16}. \quad (4.33)$$

where $\Delta = (8 + \sqrt{73 + 48\nu})/4$ is the conformal dimension. One notes that an additional shift is present in the ground state. Its existence may be due to the fact that we are considering only embeddings compatible with the kappa symmetry, rather than supersymmetry. This result differs from the results found in recent papers [3, 7]. A complete study of the kappa symmetry condition may uncover D7-brane embeddings with a vanishing shift. If not it may give us clues for its origin.

5 D5 probe brane scalar fluctuations and mesons

In this section we consider the kappa symmetric D5 probe brane embedding and analyse the corresponding spectrum of the scalar fluctuations. The considerations conceptually repeats those in the previous section.

5.1 Fluctuations along α , θ and ϕ

In order to find the fluctuations of the D5-brane we use the embedding from subsection 3.2 and the pullbacks of the fields as (2.17) and (2.19). One can make the convenient shift $\phi \rightarrow \phi + \beta = \text{const}$ in eq. (3.22) to consider only constant fluctuations along ϕ . Therefore the D5 probe brane fluctuation ansatz can be written by

$$\phi_1 = \frac{\pi}{2} + \eta \Phi_1, \quad \theta = 0 + \eta \Theta, \quad \alpha = \frac{\pi}{2} + \eta \mathcal{A}, \quad \phi = 0 + \eta \Phi, \quad (5.1)$$

where ϕ_1 , α , θ , ϕ are normal directions to the brane's world volume, and $\eta = 2\pi\alpha'$. In this case the fluctuation equations for \mathcal{A} , Θ and Φ have the same form

$$-\partial_\tau^2 \mathcal{P} + \cosh^2 \rho \tilde{\Delta}_\rho \mathcal{P} + \coth^2 \rho \Delta_{\phi_2 \phi_3} \mathcal{P} + 3 \cosh^2 \rho \tilde{\Delta}_{\beta\gamma} \mathcal{P} = 0, \quad (5.2)$$

where \mathcal{P} stands for any of the scalar fields \mathcal{A} , Θ or Φ . The differential operators in eq. (5.2) are given by

$$\tilde{\Delta}_{\beta\gamma}\mathcal{P} = \frac{3}{4}\partial_\beta^2\mathcal{P} + \partial_\gamma^2\mathcal{P}, \quad (5.3)$$

$$\tilde{\Delta}_\rho\mathcal{P} = \partial_\rho^2\mathcal{P} + (2\coth\rho + \tanh\rho)\partial_\rho\mathcal{P}, \quad (5.4)$$

$$\Delta_{\phi_2\phi_3}\mathcal{P} = \frac{1}{\sin\phi_2}\partial_{\phi_2}(\sin\phi_2\partial_{\phi_2}\mathcal{P}) + \frac{1}{\sin^2\phi_2}\partial_{\phi_3}^2\mathcal{P}. \quad (5.5)$$

Separation of variables, $\mathcal{P} = e^{i\omega\tau}R(\rho)\mathcal{Y}^\ell(\phi_2, \phi_3)Z(\beta, \gamma)$, leads to the following spectral equations:

$$\ddot{T}(\tau) = -\omega^2 T(\tau), \quad \frac{\Delta_{\phi_2\phi_3}\mathcal{Y}^\ell(\phi_2, \phi_3)}{\mathcal{Y}^\ell(\phi_2, \phi_3)} = -\ell(\ell+1), \quad \frac{\tilde{\Delta}_{\beta\gamma}Z(\beta, \gamma)}{Z(\beta, \gamma)} = -\nu, \quad (5.6)$$

$$R''(\rho) + (2\coth\rho + \tanh\rho)R'(\rho) + \left(\frac{\omega^2}{\cosh^2\rho} - \frac{\ell(\ell+1)}{\sinh^2\rho} - 3\nu\right)R(\rho) = 0. \quad (5.7)$$

It is more convenient to change the radial coordinate to $r = \sinh\rho$, after which equation (5.7) takes the form

$$R''(r) + \frac{2+4r^2}{r(r^2+1)}R'(r) + \left(\frac{\omega^2}{(r^2+1)^2} - \frac{\ell(\ell+1)}{r^2(r^2+1)} - \frac{3\nu}{r^2+1}\right)R(r) = 0. \quad (5.8)$$

The regular solution to this equation is given by

$$R(r) = r^\ell (r^2+1)^{-\frac{\omega}{2}} {}_2F_1\left(a, b; \ell + \frac{3}{2}; -r^2\right), \quad (5.9)$$

where

$$a = \frac{1}{4}(2\ell - \sqrt{12\nu+9} - 2\omega + 3), \quad b = \frac{1}{4}(2\ell + \sqrt{12\nu+9} - 2\omega + 3).$$

Imposing normalizability $b = -n$ one finds for the meson spectrum:

$$\omega = \frac{1}{2}\sqrt{12\nu+9} + \ell + \frac{3}{2} + 2n. \quad (5.10)$$

The solution near the boundary $r \rightarrow \infty$ is determined by the asymptotic equation

$$R''(r) + \frac{4}{r}R'(r) - \frac{3\nu}{r^2}R(r) = 0. \quad (5.11)$$

with the following solution

$$R(r) = c_1 r^{\frac{1}{2}(-\sqrt{12\nu+9}-3)} + c_2 r^{\frac{1}{2}(\sqrt{12\nu+9}-3)} = c_1 r^{k_2} + c_2 r^{k_1}, \quad (5.12)$$

Therefore one can calculate the conformal dimension

$$\Delta = 2 + \frac{k_1 - k_2}{2} = 2 + \frac{1}{2}\sqrt{12\nu+9}. \quad (5.13)$$

Finally, the spectrum (5.10) takes the form

$$\omega = \Delta - \frac{1}{2} + \ell + 2n. \quad (5.14)$$

The ground state ($n, \ell = 0$) is not equal to the conformal dimension Δ , but it is shifted by $-1/2$, which implies once again that the chosen kappa symmetric D5 embedding may not be compatible with supersymmetry. Another origin of this shift is also not ruled out.

5.2 Fluctuations along ϕ_1

The equation describing the scalar fluctuations along the ϕ_1 direction is given by

$$-\partial_\tau^2 \Phi_1 + \cosh^2 \rho \hat{\Delta}_\rho \Phi_1 + \coth^2 \rho \Delta_{\phi_2 \phi_3} \Phi_1 + 3 \cosh^2 \rho \tilde{\Delta}_{\beta\gamma} \Phi_1 = 0, \quad (5.15)$$

where

$$\hat{\Delta}_\rho \Phi_1 = \partial_\rho^2 \Phi_1 + (4 \coth \rho + \tanh \rho) \partial_\rho \Phi_1. \quad (5.16)$$

The operators $\tilde{\Delta}_{\beta\gamma} \Phi_1$ and $\Delta_{\phi_2 \phi_3} \Phi_1$ are defined as in (5.3) and (5.5) respectively. The radial equation here case takes the form

$$R''(\rho) + (4 \coth \rho + \tanh \rho) R'(\rho) + \left(\frac{\omega^2}{\cosh^2 \rho} - \frac{\ell(\ell+1)}{\sinh^2 \rho} - 3\nu \right) R(\rho) = 0. \quad (5.17)$$

Changing the radial variable again to $\sinh \rho = r$ one finds

$$R''(r) + \frac{4+6r^2}{r(r^2+1)} R'(r) + \left(\frac{\omega^2}{(r^2+1)^2} - \frac{\ell(\ell+1)}{r^2(r^2+1)} - \frac{3\nu}{r^2+1} \right) R(r) = 0. \quad (5.18)$$

The regular solution is written by

$$R(r) = r^{\frac{1}{2}} \left(\sqrt{4\ell(\ell+1)+9} - 3 \right) (r^2+1)^{-\frac{\omega}{2}} {}_2F_1(a, b, c, -r^2), \quad (5.19)$$

where

$$\begin{aligned} a &= \frac{1}{4} \left(-2\omega + \sqrt{4\ell(\ell+1)+9} - \sqrt{12\nu+25} + 2 \right), \\ b &= \frac{1}{4} \left(-2\omega + \sqrt{4\ell(\ell+1)+9} + \sqrt{12\nu+25} + 2 \right), \\ c &= \frac{1}{2} \left(\sqrt{4\ell(\ell+1)+9} + 2 \right). \end{aligned}$$

Imposing normalizability, $b = -n$, one finds the form of the meson spectrum

$$\omega = \Delta - 1 + \frac{1}{2} \sqrt{4\ell(\ell+1)+9} + 2n, \quad (5.20)$$

where $\Delta = \frac{1}{2} \sqrt{12\nu+25} + 2$, and a ground state given by

$$\omega_0 = \Delta + \frac{1}{2}. \quad (5.21)$$

Once again we observe that the ground state of the spectrum is shifted.

6 Conclusion

Quantum chromodynamics is the most successful theory describing the strong nuclear force so far. In the low energy regime of the theory the usual perturbative techniques are not applicable, which forces us to look for alternative non-perturbative methods. Such alternative techniques arise in string theory in the context of the AdS/CFT correspondence, where the physics of the supersymmetric Yang-Mills systems – the theory giving the best

approach to QCD – can be understood by that of the D-brane dynamics and vice versa. Considering D-brane embeddings of various dimensionality in Pilch-Warner background is important due to the fact that the holographic dual of PW background is the non-trivial $\mathcal{N} = 1$ supersymmetric fixed point of $\mathcal{N} = 4$ Yang-Mills theory. The significantly reduced supersymmetry of the dual theory can bring the duality closer to realistic QCD-like models.

In this study we considered the D5- and D7-brane embeddings compatible with the kappa symmetry preserving condition and the brane equations of motion. Working in the case when the number N_f of the flavour branes is much smaller than the number of the color branes N_c allows to neglect the backreaction of the background. The analysis of the scalar fluctuations of the probe branes and the derivation of the spectra are given analytically.

We found the explicit form of the kappa symmetry matrix for both the D5 and D7 probe branes. By solving the kappa symmetry preserving condition we were able to find brane embeddings compatible with the kappa symmetry and the classical equations of motion. We were also able to prove that the kappa symmetry preserved exactly the half of the spinor degrees of freedom for both D5 and D7.

We considered the kappa symmetric D7-brane embedding and study the corresponding spectrum of scalar fluctuations. We showed that the ground state of the spectrum along the ϕ direction is equal to the conformal dimension. However, an unexpected shift appeared in the ground state of the spectrum along the θ direction. The consideration of both kappa symmetric and supersymmetric brane embeddings may cancel the shift, but another reason for its existence should not be ruled out.

Finally, we analyzed the spectrum of the D5-brane fluctuations. Once again we considered a kappa symmetric D5 embedding. This time shifts appeared in the ground states of the spectra along all of the transverse directions. The origin of the shifts in ground states remains unclear.

All obtained spectra are equidistant, but some of their ground states are not equal to the conformal dimension of the operators dual to the fluctuations due to the presence of unexpected shifts. This result differs from the one already found in [3] and [7]. A reason for the existence of the shifts may be that the kappa symmetric brane embeddings may not be compatible with the supersymmetry. To rule out this one has to conduct a complete study of the symmetries. In any case the resolution of this puzzle is an interesting work.

It is known for a long time [26] that the Pilch-Warner supergravity is dual to the $\mathcal{N} = 4$ SYM softly broken down to $\mathcal{N} = 2$. In general adding masses to all chiral multiplets in $\mathcal{N} = 4$ SYM breaks the SUSY to $\mathcal{N} = 1$. However it can be enhanced to $\mathcal{N} = 2$. The enhancement locus in the dual geometry lies at $\theta = \pi/2$. Recent studies [15–17] present interesting results and uncover the nice structure on both sides of the AdS/CFT correspondence in this setup. It would be interesting to conduct the above analysis to this case. Initial considerations show that the equations for the fluctuations are entangled, so more thorough analysis is needed. Work on those issues is in progress and we hope to report on it soon.

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A Explicit form of the R-R and NS-NS potentials

The dilaton/axion system is trivial along the flow, i.e. $C_0 = C_8 = 0$. The C_2 potential takes the following form

$$\begin{aligned} C_2 = \Re(A_2) = & C_{\alpha\beta} d\alpha \wedge d\beta + C_{\alpha\gamma} d\alpha \wedge d\gamma + C_{\alpha\phi} d\alpha \wedge d\phi + \\ & + C_{\beta\gamma} d\beta \wedge d\gamma + C_{\gamma\phi} d\gamma \wedge d\phi + C_{\theta\alpha} d\theta \wedge d\alpha + C_{\theta\gamma} d\theta \wedge d\gamma, \end{aligned} \quad (A.1)$$

where its components in global PW coordinates are given by

$$\begin{aligned} C_{\alpha\beta} &= K \sin \beta \sin 2\theta, \quad C_{\alpha\gamma} = K \cos \alpha \sin \beta \sin 2\theta, \quad C_{\alpha\phi} = 2 K \sin \beta \sin 2\theta, \\ C_{\beta\gamma} &= K \sin \alpha \cos \beta \sin 2\theta, \quad C_{\gamma\phi} = -2 K \sin \alpha \cos \beta \sin 2\theta, \quad C_{\theta\alpha} = K (\cos 2\theta - 3) \cos \beta, \\ C_{\theta\gamma} &= K (\cos 2\theta - 3) \sin \alpha \sin \beta, \quad K = -\frac{2 \sqrt[3]{2} L^2 \cos \theta}{9 (\cos 2\theta - 3)}. \end{aligned}$$

The NS two-form

$$\begin{aligned} B_2 = \Im(A_2) = & B_{\alpha\beta} d\alpha \wedge d\beta + B_{\alpha\gamma} d\alpha \wedge d\gamma + B_{\alpha\phi} d\alpha \wedge d\phi + \\ & + B_{\beta\gamma} d\beta \wedge d\gamma + B_{\gamma\phi} d\gamma \wedge d\phi + B_{\theta\alpha} d\theta \wedge d\alpha + B_{\theta\gamma} d\theta \wedge d\gamma \end{aligned} \quad (A.2)$$

has components

$$\begin{aligned} B_{\alpha\beta} &= B_0 \cos \beta \sin 2\theta, \quad B_{\alpha\gamma} = B_0 \cos \alpha \cos \beta \sin 2\theta, \quad B_{\alpha\phi} = 2 B_0 \cos \beta \sin 2\theta, \\ B_{\beta\gamma} &= -B_0 \sin \alpha \sin \beta \sin 2\theta, \quad B_{\gamma\phi} = 2 B_0 \sin \alpha \sin \beta \sin 2\theta, \quad B_{\theta\alpha} = -B_0 (\cos 2\theta - 3) \sin \beta, \\ B_{\theta\gamma} &= B_0 (\cos 2\theta - 3) \sin \alpha \cos \beta, \quad B_0 = -K. \end{aligned}$$

We also have expressions for the four-form potential C_4 :

$$C_4 = \frac{2^{5/3} L^4 \sinh^4 \rho \sin^2 \phi_1 \sin \phi_2}{3} d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3, \quad (A.3)$$

and its dual 4-form:

$$\tilde{C}_4 = -\frac{2^{5/3} L^4 (\cos 4\theta - 6 \cos 2\theta + 16 (\cos 2\theta - 3) \ln(3 - \cos 2\theta) - 31) \sin \alpha}{81 (\cos 2\theta - 3)} d\alpha \wedge d\beta \wedge d\gamma \wedge d\phi. \quad (A.4)$$

The six-form potential C_6 has more complicated structure given by

$$\begin{aligned}
C_6 = & -\frac{16 L^6 \varepsilon_\phi \sinh^4 \rho \sin 2\theta \cos \theta \cos \beta}{27 (\cos 2\theta - 3)} d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\alpha \wedge d\phi \\
& -\frac{16 L^6 \varepsilon_\phi \sinh^4 \rho \sin 2\theta \cos \theta \sin \alpha \sin \beta}{27 (\cos 2\theta - 3)} d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\gamma \wedge d\phi \\
& -\frac{L^6 \varepsilon_\phi}{9} \sinh^4 \rho \cos \theta (\cos 2\theta - 3) \sin \beta d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\theta \wedge d\alpha \\
& +\frac{L^6 \varepsilon_\phi}{9} \sinh^4 \rho \cos \theta (\cos 2\theta - 3) \sin \alpha \cos \beta d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\theta \wedge d\gamma \\
& +\frac{L^6 \varepsilon_\phi \sinh^4 \rho \sin 2\theta \cos \theta (\cos 2\theta - 11) \cos \beta}{27 (\cos 2\theta - 3)} d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\alpha \wedge d\beta \\
& +\frac{L^6 \varepsilon_\phi \sinh^4 \rho \sin 2\theta \cos \theta (\cos 2\theta - 11) \cos \alpha \cos \beta}{27 (\cos 2\theta - 3)} d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\alpha \wedge d\gamma \\
& -\frac{L^6 \varepsilon_\phi \sinh^4 \rho \sin 2\theta \cos \theta (\cos 2\theta - 11) \sin \alpha \sin \beta}{27 (\cos 2\theta - 3)} d\tau \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\beta \wedge d\gamma,
\end{aligned} \tag{A.5}$$

where $\varepsilon_\phi = \sin^2 \phi_1 \sin \phi_2$. All the R-R and NS-NS potentials are written explicitly in global coordinates and satisfy the corresponding Bianchi identities and equations of motion (2.10).

B D=10 Dirac gamma matrices

In order to calculate the kappa symmetry matrix for the corresponding D-branes one has to chose an appropriate basis for the 10-dimensional Dirac gamma matrices Γ_μ . We choose to work with the Majorana-Weyl representation:

$$\begin{aligned}
\Gamma_0 &= i \sigma_1 \otimes 1 \otimes 1 \otimes 1 \otimes 1, & \Gamma_1 &= i \chi \otimes \chi \otimes \chi \otimes \chi \otimes \chi, \\
\Gamma_2 &= i \chi \otimes \chi \otimes 1 \otimes \sigma_1 \otimes \chi, & \Gamma_3 &= i \chi \otimes \chi \otimes 1 \otimes \sigma_3 \otimes \chi, \\
\Gamma_4 &= i \chi \otimes \chi \otimes \sigma_1 \otimes \chi \otimes 1, & \Gamma_5 &= i \chi \otimes \chi \otimes \sigma_3 \otimes \chi \otimes 1, \\
\Gamma_6 &= i \chi \otimes \chi \otimes \chi \otimes 1 \otimes \sigma_1, & \Gamma_7 &= i \chi \otimes \chi \otimes \chi \otimes 1 \otimes \sigma_3, \\
\Gamma_8 &= i \chi \otimes \sigma_1 \otimes 1 \otimes 1 \otimes 1, & \Gamma_9 &= i \chi \otimes \sigma_3 \otimes 1 \otimes 1 \otimes 1,
\end{aligned}$$

$$\Gamma_* = \sigma_3 \otimes 1 \otimes 1 \otimes 1 \otimes 1 = \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 \Gamma_6 \Gamma_7 \Gamma_8 \Gamma_9 = \begin{pmatrix} 1_{16 \times 16} & 0 \\ 0 & -1_{16 \times 16} \end{pmatrix},$$

where $\chi = i \sigma_2$, and σ_i are the Pauli matrices:

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{B.1}$$

All the gamma matrices satisfy the Clifford algebra

$$\{\Gamma_\mu, \Gamma_\nu\} = 2 \eta_{\mu\nu} 1_{32 \times 32}, \quad \eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1). \tag{B.2}$$

The rule for the tensor product is the one used by Mathematica (the right matrix goes into the left one):

$$a \otimes b = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} b_{11} & a_{11} b_{12} & a_{12} b_{11} & a_{12} b_{12} \\ a_{11} b_{21} & a_{11} b_{22} & a_{12} b_{21} & a_{12} b_{22} \\ a_{21} b_{11} & a_{21} b_{12} & a_{22} b_{11} & a_{22} b_{12} \\ a_{21} b_{21} & a_{21} b_{22} & a_{22} b_{21} & a_{22} b_{22} \end{pmatrix}.$$

This assures the correct form of $\Gamma_* = \text{diag}(1, \dots, 1, -1, \dots, -1)$.

C Vielbein coefficients

The induced gamma matrices on the world volume are given by

$$\gamma_a = \partial_a X^M E_M^I \Gamma_I, \quad (\text{C.1})$$

where E_M^I , $I = 0, \dots, 9$, are the vielbein coefficients of the PW metric

$$ds^2 = \eta_{IJ} e^I e^J. \quad (\text{C.2})$$

The basis one-forms $e^I = E_M^I dX^M$ take the form:

$$\begin{aligned} e^0 &= i L \Omega \cosh \rho d\tau = E_0^0 d\tau, \\ e^1 &= L \Omega d\rho = E_1^1 d\rho, \\ e^2 &= L \Omega \sinh \rho d\phi_1 = E_2^2 d\phi_1, \\ e^3 &= L \Omega \sinh \rho \sin \phi_1 d\phi_2 = E_3^3 d\phi_2, \\ e^4 &= L \Omega \sinh \rho \sin \phi_1 \sin \phi_2 d\phi_3 = E_4^4 d\phi_3, \\ e^5 &= \sqrt{\frac{2}{3}} L \Omega d\theta = E_5^5 d\theta, \\ e^6 &= \sqrt{\frac{2}{3}} L \Omega \frac{2 \cos \theta}{\sqrt{3 - \cos 2\theta}} \sigma_1 = E_6^6 d\alpha + E_8^6 d\gamma, \\ e^7 &= \sqrt{\frac{2}{3}} L \Omega \frac{2 \cos \theta}{\sqrt{3 - \cos 2\theta}} \sigma_2 = E_6^7 d\alpha + E_8^7 d\gamma, \\ e^8 &= \sqrt{\frac{2}{3}} L \Omega \frac{2 \sin 2\theta}{(3 - \cos 2\theta)} (\sigma_3 + d\phi) = E_7^8 d\beta + E_8^8 d\gamma + E_9^8 d\phi, \\ e^9 &= \frac{2}{3} L \Omega \left(\frac{1 - 3 \cos 2\theta}{\cos 2\theta - 3} \right) \left(d\phi - \frac{4 \cos^2 \theta}{1 - 3 \cos 2\theta} \sigma_3 \right) = E_7^9 d\beta + E_8^9 d\gamma + E_9^9 d\phi, \end{aligned}$$

where

$$\begin{aligned} \sigma_1 &= \frac{1}{2} (\sin \beta d\alpha - \cos \beta \sin \alpha d\gamma), \\ \sigma_2 &= -\frac{1}{2} (\cos \beta d\alpha + \sin \beta \sin \alpha d\gamma), \\ \sigma_3 &= \frac{1}{2} (d\beta + \cos \alpha d\gamma), \end{aligned}$$

are the left-invariant one forms. The expressions for the vielbein coefficients are written by

$$\begin{aligned}
E_0^0 &= E_\tau^0 = i L \Omega \cosh \rho, \\
E_1^1 &= E_\rho^1 = L \Omega, \\
E_2^2 &= E_{\phi_1}^2 = L \Omega \sinh \rho, \\
E_3^3 &= E_{\phi_2}^3 = L \Omega \sinh \rho \sin \phi_1, \\
E_4^4 &= E_{\phi_3}^4 = L \Omega \sinh \rho \sin \phi_1 \sin \phi_2, \\
E_5^5 &= E_\theta^5 = \sqrt{\frac{2}{3}} L \Omega, \\
E_6^6 &= E_\alpha^6 = \frac{1}{2} \sqrt{\frac{2}{3}} L \Omega \frac{2 \cos \theta \sin \beta}{\sqrt{3 - \cos 2\theta}}, \\
E_8^8 &= E_\gamma^8 = \frac{1}{2} \sqrt{\frac{2}{3}} L \Omega \frac{2 \sin 2\theta \cos \alpha}{(3 - \cos 2\theta)}, \\
E_9^9 &= E_\phi^9 = \frac{2}{3} L \Omega \left(\frac{1 - 3 \cos 2\theta}{\cos 2\theta - 3} \right), \\
E_8^6 &= E_\gamma^6 = -\frac{1}{2} \sqrt{\frac{2}{3}} L \Omega \frac{2 \cos \theta \cos \beta \sin \alpha}{\sqrt{3 - \cos 2\theta}}, \\
E_6^7 &= E_\alpha^7 = -\frac{1}{2} \sqrt{\frac{2}{3}} L \Omega \frac{2 \cos \theta \cos \beta}{\sqrt{3 - \cos 2\theta}}, \\
E_8^7 &= E_\gamma^7 = -\frac{1}{2} \sqrt{\frac{2}{3}} L \Omega \frac{2 \cos \theta \sin \beta \sin \alpha}{\sqrt{3 - \cos 2\theta}}, \\
E_7^8 &= E_\beta^8 = \frac{1}{2} \sqrt{\frac{2}{3}} L \Omega \frac{2 \sin 2\theta}{(3 - \cos 2\theta)}, \\
E_9^8 &= E_\phi^8 = \sqrt{\frac{2}{3}} L \Omega \frac{2 \sin 2\theta}{(3 - \cos 2\theta)}, \\
E_7^9 &= E_\beta^9 = -\frac{1}{3} L \Omega \left(\frac{1 - 3 \cos 2\theta}{\cos 2\theta - 3} \right) \frac{4 \cos^2 \theta}{1 - 3 \cos 2\theta}, \\
E_8^9 &= E_\gamma^9 = -\frac{1}{3} L \Omega \left(\frac{1 - 3 \cos 2\theta}{\cos 2\theta - 3} \right) \frac{4 \cos^2 \theta \cos \alpha}{1 - 3 \cos 2\theta},
\end{aligned}$$

where Ω is the warp factor from eq. (2.9), and L is the AdS radius.

D Explicit form of the D7 κ symmetry matrix

The 8-form κ symmetry matrix for the D7-brane has the following form

$$(\Gamma_\kappa)_{(8)} = \frac{(M_8 \sigma_3^4 + M_6 \sigma_3^3 + M_4 \sigma_3^2) i \sigma_2}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} d\tau \wedge d\rho \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\alpha \wedge d\beta \wedge d\gamma, \quad (\text{D.1})$$

where the Pauli matrices acts on the two component spinor $(\varepsilon_1^\alpha, \varepsilon_2^\beta)$, $\alpha, \beta = 1, \dots, 16$. The explicit form of the M_4 matrix is given by

$$\begin{aligned}
M_4 &= a_1 \gamma_{\rho\phi_2\phi_3\beta} + a_2 \gamma_{\rho\phi_1\phi_3\beta} + a_3 \gamma_{\rho\phi_1\phi_2\beta} + a_4 \gamma_{\phi_1\phi_2\phi_3\beta} + a_5 \gamma_{\tau\phi_2\phi_3\beta} + a_6 \gamma_{\tau\phi_1\phi_3\beta} \\
&+ a_7 \gamma_{\tau\phi_1\phi_2\beta} + a_8 \gamma_{\tau\rho\phi_3\beta} + a_9 \gamma_{\tau\rho\phi_2\beta} + a_{10} \gamma_{\tau\rho\phi_1\beta} + a_{11} \gamma_{\rho\phi_1\phi_2\phi_3} + a_{12} \gamma_{\tau\phi_1\phi_2\phi_3} \\
&+ a_{13} \gamma_{\tau\rho\phi_2\phi_3} + a_{14} \gamma_{\tau\rho\phi_1\phi_3} + a_{15} \gamma_{\tau\rho\phi_1\phi_2}, \quad (\text{D.2})
\end{aligned}$$

where $\gamma_{\rho\phi_1\phi_2\beta} = 4! \gamma_{[\rho\phi_1\phi_2\beta]}$ is an antisymmetric product of induced gamma matrices, and

$$\begin{aligned}
a_0 &= B_{\gamma\phi} B_{\theta\alpha} - B_{\alpha\phi} B_{\theta\gamma}, \\
a_1 &= -a_0 (\partial_\tau \phi \partial_{\phi_1} \theta - \partial_\tau \theta \partial_{\phi_1} \phi), \quad a_2 = a_0 (\partial_\tau \phi \partial_{\phi_2} \theta - \partial_\tau \theta \partial_{\phi_2} \phi), \\
a_3 &= -a_0 (\partial_\tau \phi \partial_{\phi_3} \theta - \partial_\tau \theta \partial_{\phi_3} \phi), \quad a_4 = -a_0 (\partial_\rho \phi \partial_\tau \theta - \partial_\rho \theta \partial_\tau \phi),
\end{aligned}$$

$$\begin{aligned}
a_5 &= a_0 (\partial_\rho \phi \partial_{\phi_1} \theta - \partial_\rho \theta \partial_{\phi_1} \phi), & a_6 &= -a_0 (\partial_\rho \phi \partial_{\phi_2} \theta - \partial_\rho \theta \partial_{\phi_2} \phi), \\
a_7 &= a_0 (\partial_\rho \phi \partial_{\phi_3} \theta - \partial_\rho \theta \partial_{\phi_3} \phi), & a_8 &= a_0 (\partial_{\phi_1} \phi \partial_{\phi_2} \theta - \partial_{\phi_1} \theta \partial_{\phi_2} \phi), \\
a_9 &= -a_0 (\partial_{\phi_1} \phi \partial_{\phi_3} \theta - \partial_{\phi_1} \theta \partial_{\phi_3} \phi), & a_{10} &= a_0 (\partial_{\phi_2} \phi \partial_{\phi_3} \theta - \partial_{\phi_2} \theta \partial_{\phi_3} \phi), \\
a_{11} &= B_{\beta\gamma} (B_{\theta\alpha} \partial_\tau \theta - B_{\alpha\phi} \partial_\tau \phi) + B_{\alpha\beta} (B_{\theta\gamma} \partial_\tau \theta - B_{\gamma\phi} \partial_\tau \phi) - a_0 (\partial_\beta \phi \partial_\tau \theta - \partial_\beta \theta \partial_\tau \phi), \\
a_{12} &= B_{\beta\gamma} (-B_{\theta\alpha} \partial_\rho \theta + B_{\alpha\phi} \partial_\rho \phi) + B_{\alpha\beta} (-B_{\theta\gamma} \partial_\rho \theta + B_{\gamma\phi} \partial_\rho \phi) + a_0 (\partial_\beta \phi \partial_\rho \theta - \partial_\beta \theta \partial_\rho \phi), \\
a_{13} &= B_{\beta\gamma} (B_{\theta\alpha} \partial_{\phi_1} \theta - B_{\alpha\phi} \partial_{\phi_1} \phi) + B_{\alpha\beta} (B_{\theta\gamma} \partial_{\phi_1} \theta - B_{\gamma\phi} \partial_{\phi_1} \phi) - a_0 (\partial_\beta \phi \partial_{\phi_1} \theta - \partial_\beta \theta \partial_{\phi_1} \phi), \\
a_{14} &= B_{\beta\gamma} (-B_{\theta\alpha} \partial_{\phi_2} \theta + B_{\alpha\phi} \partial_{\phi_2} \phi) + B_{\alpha\beta} (-B_{\theta\gamma} \partial_{\phi_2} \theta + B_{\gamma\phi} \partial_{\phi_2} \phi) + a_0 (\partial_\beta \phi \partial_{\phi_2} \theta - \partial_\beta \theta \partial_{\phi_2} \phi), \\
a_{15} &= B_{\beta\gamma} (B_{\theta\alpha} \partial_{\phi_3} \theta - B_{\alpha\phi} \partial_{\phi_3} \phi) + B_{\alpha\beta} (B_{\theta\gamma} \partial_{\phi_3} \theta - B_{\gamma\phi} \partial_{\phi_3} \phi) - a_0 (\partial_\beta \phi \partial_{\phi_3} \theta - \partial_\beta \theta \partial_{\phi_3} \phi).
\end{aligned}$$

Here the $B_{\mu\nu}$, $\mu, \nu = 0, \dots, 7$, are the components of the second rank antisymmetric Kalb-Ramond B-field (A.2). The M_6 matrix is given by

$$\begin{aligned}
M_6 &= b_1 \gamma_{\rho\phi_1\phi_2\phi_3\alpha\beta} + b_2 \gamma_{\rho\phi_1\phi_2\phi_3\beta\gamma} + b_3 \gamma_{\tau\rho\phi_1\phi_2\alpha\beta} + b_4 \gamma_{\tau\rho\phi_1\phi_2\beta\gamma} + b_5 \gamma_{\tau\rho\phi_1\phi_2\phi_3\alpha} + b_6 \gamma_{\tau\rho\phi_1\phi_2\phi_3\beta} \\
&\quad + b_7 \gamma_{\tau\rho\phi_1\phi_2\phi_3\gamma} + b_8 \gamma_{\tau\rho\phi_1\phi_3\alpha\beta} + b_9 \gamma_{\tau\rho\phi_1\phi_3\beta\gamma} + b_{10} \gamma_{\tau\rho\phi_2\phi_3\alpha\beta} + b_{11} \gamma_{\tau\rho\phi_2\phi_3\beta\gamma} + b_{12} \gamma_{\tau\phi_1\phi_2\phi_3\alpha\beta} \\
&\quad + b_{13} \gamma_{\tau\phi_1\phi_2\phi_3\beta\gamma},
\end{aligned} \tag{D.3}$$

where

$$\begin{aligned}
b_1 &= B_{\theta\gamma} \partial_\tau \theta - B_{\gamma\phi} \partial_\tau \phi, & b_2 &= B_{\theta\alpha} \partial_\tau \theta - B_{\alpha\phi} \partial_\tau \phi, \\
b_3 &= B_{\theta\gamma} \partial_{\phi_3} \theta - B_{\gamma\phi} \partial_{\phi_3} \phi, & b_4 &= B_{\theta\alpha} \partial_{\phi_3} \theta - B_{\alpha\phi} \partial_{\phi_3} \phi, \\
b_5 &= B_{\beta\gamma} + B_{\theta\gamma} \partial_\beta \theta - B_{\gamma\phi} \partial_\beta \phi, & b_6 &= -(B_{\alpha\gamma} + B_{\theta\gamma} \partial_\alpha \theta - B_{\gamma\phi} \partial_\alpha \phi - B_{\theta\alpha} \partial_\gamma \theta + B_{\alpha\phi} \partial_\gamma \phi), \\
b_7 &= B_{\alpha\beta} - B_{\theta\alpha} \partial_\beta \theta + B_{\alpha\phi} \partial_\beta \phi, & b_8 &= -B_{\theta\gamma} \partial_{\phi_2} \theta + B_{\gamma\phi} \partial_{\phi_2} \phi, & b_9 &= -B_{\theta\alpha} \partial_{\phi_2} \theta + B_{\alpha\phi} \partial_{\phi_2} \phi, \\
b_{10} &= B_{\theta\gamma} \partial_{\phi_1} \theta - B_{\gamma\phi} \partial_{\phi_1} \phi, & b_{11} &= B_{\theta\alpha} \partial_{\phi_1} \theta - B_{\alpha\phi} \partial_{\phi_1} \phi, & b_{12} &= -B_{\theta\gamma} \partial_\rho \theta + B_{\gamma\phi} \partial_\rho \phi, \\
b_{13} &= -B_{\theta\alpha} \partial_\rho \theta + B_{\alpha\phi} \partial_\rho \phi.
\end{aligned}$$

The M_8 matrix takes the form

$$M_8 = \gamma_{\tau\rho\phi_1\phi_2\phi_3\alpha\beta\gamma}. \tag{D.4}$$

The expressions for the induced gamma matrices are

$$\begin{aligned}
\gamma_\tau &= E_\tau^0 \Gamma_0 + \partial_\tau \theta E_\theta^5 \Gamma_5 + \partial_\tau \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right), \\
\gamma_\rho &= E_\rho^1 \Gamma_1 + \partial_\rho \theta E_\theta^5 \Gamma_5 + \partial_\rho \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right), \\
\gamma_{\phi_1} &= E_{\phi_1}^2 \Gamma_2 + \partial_{\phi_1} \theta E_\theta^5 \Gamma_5 + \partial_{\phi_1} \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right), \\
\gamma_{\phi_2} &= E_{\phi_2}^3 \Gamma_3 + \partial_{\phi_2} \theta E_\theta^5 \Gamma_5 + \partial_{\phi_2} \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right), \\
\gamma_{\phi_3} &= E_{\phi_3}^4 \Gamma_4 + \partial_{\phi_3} \theta E_\theta^5 \Gamma_5 + \partial_{\phi_3} \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right), \\
\gamma_\alpha &= E_\alpha^6 \Gamma_6 + \partial_\alpha \theta E_\theta^5 \Gamma_5 + \partial_\alpha \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right), \\
\gamma_\beta &= E_\beta^8 \Gamma_8 + E_\beta^9 \Gamma_9 + \partial_\beta \theta E_\theta^5 \Gamma_5 + \partial_\beta \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right), \\
\gamma_\gamma &= E_\gamma^7 \Gamma_7 + E_\gamma^8 \Gamma_8 + E_\gamma^9 \Gamma_9 + \partial_\gamma \theta E_\theta^5 \Gamma_5 + \partial_\gamma \phi \left(E_\phi^8 \Gamma_8 + E_\phi^9 \Gamma_9 \right).
\end{aligned}$$

The kappa symmetry preserving condition (3.1) takes the form

$$\frac{1}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} \left(M_8 \begin{pmatrix} \varepsilon_2^\beta \\ -\varepsilon_1^\alpha \end{pmatrix} + M_6 \begin{pmatrix} \varepsilon_2^\beta \\ \varepsilon_1^\alpha \end{pmatrix} + M_4 \begin{pmatrix} \varepsilon_2^\beta \\ -\varepsilon_1^\alpha \end{pmatrix} \right) = \begin{pmatrix} \varepsilon_1^\alpha \\ \varepsilon_2^\beta \end{pmatrix}. \quad (\text{D.5})$$

This is an algebraic system for the components of the spinor. We can solve it by choosing a simpler D7-brane embedding ansatz, namely

$$\theta = \theta(\rho), \quad \phi = \phi(\beta). \quad (\text{D.6})$$

In this case the system (D.5) takes the form

$$\sum_{\beta=1}^{16} a_{\alpha\beta} \epsilon_2^{(\beta)} = \epsilon_1^{(\alpha)}, \quad \alpha = 1, \dots, 16, \quad (\text{D.7})$$

$$\sum_{\beta=1}^{16} b_{\alpha\beta} \epsilon_1^{(\beta)} = \epsilon_2^{(\alpha)}, \quad \alpha = 1, \dots, 16, \quad (\text{D.8})$$

where the coefficients $a_{\alpha\beta}$ and $b_{\alpha\beta}$ are functions of the coordinates. We can substitute the $\epsilon_2^{(\beta)}$ from (D.8) into (D.7). The resulting system is a homogenous system of 16 equations for the components $\epsilon_1^{(\alpha)}$:

$$\sum_{\beta=1}^{16} s_{\alpha\beta} \epsilon_1^{(\beta)} = 0, \quad \alpha = 1, \dots, 16. \quad (\text{D.9})$$

We can write it explicitly as

$$\begin{aligned} A_1 \varepsilon_1 + A_2 \varepsilon_{11} + A_3 \varepsilon_{16} &= 0, & A_1 \varepsilon_2 - A_2 \varepsilon_{12} + A_3 \varepsilon_{15} &= 0, \\ A_1 \varepsilon_3 - A_2 \varepsilon_9 - A_3 \varepsilon_{14} &= 0, & A_1 \varepsilon_4 + A_2 \varepsilon_{10} - A_3 \varepsilon_{13} &= 0, \\ A_1 \varepsilon_5 + A_2 \varepsilon_{15} - A_3 \varepsilon_{12} &= 0, & A_1 \varepsilon_6 - A_2 \varepsilon_{16} - A_3 \varepsilon_{11} &= 0, \\ A_1 \varepsilon_7 - A_2 \varepsilon_{13} - A_3 \varepsilon_{10} &= 0, & A_1 \varepsilon_8 + A_2 \varepsilon_{14} - A_3 \varepsilon_9 &= 0, \\ A_1 \varepsilon_9 + A_2 \varepsilon_3 - A_3 \varepsilon_8 &= 0, & A_1 \varepsilon_{10} + A_2 \varepsilon_4 - A_3 \varepsilon_7 &= 0, \\ A_1 \varepsilon_{11} - A_2 \varepsilon_1 + A_3 \varepsilon_6 &= 0, & A_1 \varepsilon_{12} + A_2 \varepsilon_2 - A_3 \varepsilon_5 &= 0, \\ A_1 \varepsilon_{13} - A_2 \varepsilon_7 - A_3 \varepsilon_4 &= 0, & A_1 \varepsilon_{14} + A_2 \varepsilon_8 - A_3 \varepsilon_3 &= 0, \\ A_1 \varepsilon_{15} + A_2 \varepsilon_5 - A_3 \varepsilon_2 &= 0, & A_1 \varepsilon_{16} - A_2 \varepsilon_6 - A_3 \varepsilon_1 &= 0, \end{aligned}$$

where

$$\begin{aligned} A_1 = & -D_0 - 37324800 (\partial_\rho \theta)^2 \cos^2 \beta \cos^2 \theta (3 - \cos[2\theta])^{7/2} (1 - \partial_\beta \phi + (1 + 3 \partial_\beta \phi) \cos 2\theta)^2 \sin^2 \alpha \\ & - 82944 (1 + 2 \partial_\beta \phi)^2 (\partial_\rho \theta)^2 \cos^4 \theta (3 - \cos 2\theta)^{7/2} \sin^2 \alpha \sin^2 \theta \\ & - 223948800 (1 + 2 \partial_\beta \phi)^2 (\partial_\rho \theta)^2 \cos^2 \beta \cos^4 \theta (3 - \cos 2\theta)^{7/2} \sin^2 \alpha \sin^2 \theta \\ & - 335923200 (1 + 2 \partial_\beta \phi)^2 \cos^4 \theta (3 - \cos 2\theta)^{7/2} \sin^2 \alpha \sin^2 \beta \sin^2 \theta \\ & - 223948800 (1 + 2 \partial_\beta \phi)^2 (\partial_\rho \theta)^2 \cos^4 \theta (3 - \cos 2\theta)^{7/2} \sin^2 \alpha \sin^2 \beta \sin^2 \theta \\ & - 55987200 \partial_\beta \phi^2 \cos^2 \alpha \left(-56 \sqrt{2} \partial_\rho \theta + \cos \beta \right)^2 (3 - \cos 2\theta)^{9/2} \sin^2 2\theta \\ & - 18662400 \partial_\beta \phi^2 \cos^2 \alpha \left(168 + \sqrt{2} \partial_\rho \theta \cos \beta \right)^2 \cos^2 \theta (3 - \cos 2\theta)^{5/2} (-7 \sin \theta + \sin 3\theta)^2 \\ & - 37324800 \partial_\beta \phi^2 (\partial_\rho \theta)^2 \cos^2 \alpha \cos^2 \theta (3 - \cos 2\theta)^{5/2} \sin^2 \beta (-7 \sin \theta + \sin 3\theta)^2, \end{aligned}$$

$$A_2 = -\sqrt{\frac{2}{3}} \partial_\rho \theta A_3, \quad D_0 = \frac{729 \sqrt{3} (3 - \cos 2\theta)^{7/4}}{4 \sqrt[3]{2} L^8 \sinh^3 \rho \cosh \rho \sin^2 \phi_1 \sin \phi_2} \sqrt{-\det(\mathcal{G} + \mathcal{B})},$$

$$A_3 = 64696320 \sqrt{6} (1 + 2 \partial_\beta \phi) \cos^3 \theta (-3 + \cos 2\theta)^4 \sin 2\alpha \sin^2 \theta \partial_\beta \phi (\partial_\rho \theta)^2,$$

A homogenous system has a non-trivial solution when the determinant of its matrix is equal to zero. In our case we have to impose

$$\det(s_{\alpha\beta}) = \frac{\left(-3 A_1^2 + A_3^2 \left(3 + 2 (\partial_\rho \theta)^2\right)\right)^8}{6561} = 0. \quad (\text{D.10})$$

This allows us to find the following non-trivial solution of (D.9):

$$\begin{aligned} \varepsilon_9^{(1)} &= \frac{-\sqrt{2} \partial_\rho \theta \varepsilon_3^{(1)} + \sqrt{3} \varepsilon_8^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}, & \varepsilon_{10}^{(1)} &= \frac{\sqrt{2} \partial_\rho \theta \varepsilon_4^{(1)} + \sqrt{3} \varepsilon_7^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}, \\ \varepsilon_{11}^{(1)} &= \frac{\sqrt{2} \partial_\rho \theta \varepsilon_1^{(1)} - \sqrt{3} \varepsilon_6^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}, & \varepsilon_{12}^{(1)} &= \frac{-\sqrt{2} \partial_\rho \theta \varepsilon_2^{(1)} - \sqrt{3} \varepsilon_5^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}, \\ \varepsilon_{13}^{(1)} &= \frac{\sqrt{3} \varepsilon_4^{(1)} - \sqrt{2} \partial_\rho \theta \varepsilon_7^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}, & \varepsilon_{14}^{(1)} &= \frac{\sqrt{3} \varepsilon_3^{(1)} + \sqrt{2} \partial_\rho \theta \varepsilon_8^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}, \\ \varepsilon_{15}^{(1)} &= \frac{-\sqrt{3} \varepsilon_2^{(1)} + \sqrt{2} \partial_\rho \theta \varepsilon_5^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}, & \varepsilon_{16}^{(1)} &= \frac{-\sqrt{3} \varepsilon_1^{(1)} - \sqrt{2} \partial_\rho \theta \varepsilon_6^{(1)}}{\sqrt{3 + 2 (\partial_\rho \theta)^2}}. \end{aligned}$$

Setting $\theta = \text{const} = 0$ one finds

$$\begin{aligned} \varepsilon_9^{(1)} &= \varepsilon_8^{(1)}, & \varepsilon_{10}^{(1)} &= \varepsilon_7^{(1)}, & \varepsilon_{11}^{(1)} &= -\varepsilon_6^{(1)}, & \varepsilon_{12}^{(1)} &= -\varepsilon_5^{(1)}, \\ \varepsilon_{13}^{(1)} &= -\varepsilon_7^{(1)}, & \varepsilon_{14}^{(1)} &= \varepsilon_8^{(1)}, & \varepsilon_{15}^{(1)} &= \varepsilon_5^{(1)}, & \varepsilon_{16}^{(1)} &= -\varepsilon_6^{(1)}. \end{aligned} \quad (\text{D.11})$$

There are 8 non-zero spinor solutions. This is what we expected – the kappa symmetry removed exactly half of the spinor components. Setting $\theta = 0$ in (D.10) also gives us $1 + \partial_\beta \phi = 0$, which is solved by $\phi = -\beta + c$.

E Explicit form of the D5 κ symmetry matrix

The D5 6-form kappa symmetry matrix is given by

$$(\Gamma_\kappa)_{(6)} = \frac{M_2 i \sigma_3^1 \sigma_2 + M_4 i \sigma_3^2 \sigma_2}{\sqrt{-\det(\mathcal{G} + \mathcal{B})}} d\tau \wedge d\rho \wedge d\phi_2 \wedge d\phi_3 \wedge d\beta \wedge d\gamma, \quad (\text{E.1})$$

where we can write the matrices M_2 and M_4 as

$$M_2 = \sum_{i=1}^{15} A_i a_i, \quad M_4 = \sum_{i=1}^{15} C_i c_i. \quad (\text{E.2})$$

The matrices A_i are given by the following products of the induced gamma matrices:

$$\begin{aligned} A_1 &= \gamma_{\beta\gamma}, & A_2 &= \gamma_{\rho\beta}, & A_3 &= \gamma_{\rho\gamma}, & A_4 &= \gamma_{\rho\phi_2}, & A_5 &= \gamma_{\rho\phi_3}, \\ A_6 &= \gamma_{\tau\beta}, & A_7 &= \gamma_{\tau\gamma}, & A_8 &= \gamma_{\tau\rho}, & A_9 &= \gamma_{\tau\phi_2}, & A_{10} &= \gamma_{\tau\phi_3}, \\ A_{11} &= \gamma_{\phi_2\beta}, & A_{12} &= \gamma_{\phi_2\gamma}, & A_{13} &= \gamma_{\phi_2\phi_3}, & A_{14} &= \gamma_{\phi_3\beta}, & A_{15} &= \gamma_{\phi_3\gamma}. \end{aligned}$$

The coefficients a_i are expressed by the components of the induced B -field:

$$\begin{aligned} a_1 &= -b_{\rho\phi_3} b_{\tau\phi_2} + b_{\rho\phi_2} b_{\tau\phi_3} + b_{\tau\rho} b_{\phi_2\phi_3}, & a_2 &= -b_{\tau\phi_3} b_{\phi_2\gamma} + b_{\tau\gamma} b_{\phi_2\phi_3} + b_{\tau\phi_2} b_{\phi_3\gamma}, \\ a_3 &= b_{\tau\phi_3} b_{\phi_2\beta} - b_{\tau\beta} b_{\phi_2\phi_3} - b_{\tau\phi_2} b_{\phi_3\beta}, & a_4 &= b_{\beta\gamma} b_{\tau\phi_3} + b_{\tau\gamma} b_{\phi_3\beta} - b_{\tau\beta} b_{\phi_3\gamma}, \\ a_5 &= b_{\beta\gamma} b_{\tau\phi_2} + b_{\tau\gamma} b_{\phi_2\beta} - b_{\tau\beta} b_{\phi_2\gamma}, & a_6 &= b_{\rho\phi_3} b_{\phi_2\gamma} - b_{\rho\gamma} b_{\phi_2\phi_3} - b_{\rho\phi} b_{\phi_3\gamma}, \\ a_7 &= -b_{\rho\phi_3} b_{\phi_2\beta} + b_{\rho\beta} b_{\phi_2\phi_3} + b_{\rho\phi_2} b_{\phi_3\beta}, & a_8 &= b_{\beta\gamma} b_{\phi_2\phi_3} + b_{\phi_2\gamma} b_{\phi_3\beta} - b_{\phi_2\beta} b_{\phi_3\gamma}, \\ a_9 &= b_{\beta\gamma} b_{\rho\phi_3} + b_{\rho\gamma} b_{\phi_3\beta} - b_{\rho\beta} b_{\phi_3\gamma}, & a_{10} &= b_{\beta\gamma} b_{\rho\phi_2} + b_{\rho\gamma} b_{\phi_2\beta} - b_{\rho\beta} b_{\phi_2\gamma}, \\ a_{12} &= b_{\rho\phi_3} b_{\tau\beta} - b_{\rho\beta} b_{\tau\phi_3} + b_{\tau\rho} b_{\phi_3\beta}, & a_{13} &= -b_{\rho\gamma} b_{\tau\beta} + b_{\rho\beta} b_{\tau\gamma} + b_{\beta\gamma} b_{\tau\rho}, \\ a_{14} &= b_{\rho\phi_2} b_{\tau\gamma} - b_{\rho\gamma} b_{\tau\phi_2} + b_{\tau\rho} b_{\phi_2\gamma}, & a_{15} &= b_{\rho\phi_2} b_{\tau\beta} - b_{\rho\beta} b_{\tau\phi_2} + b_{\tau\rho} b_{\phi_2\beta}. \end{aligned}$$

The expressions for the matrices C_i are

$$\begin{aligned} C_1 &= \gamma_{\rho\phi_2\beta\gamma}, & C_2 &= \gamma_{\rho\phi_2\phi_3\beta}, & C_3 &= \gamma_{\rho\phi_2\phi_3\gamma}, & C_4 &= \gamma_{\rho\phi_3\beta\gamma}, & C_5 &= \gamma_{\tau\rho\beta\gamma}, \\ C_6 &= \gamma_{\tau\rho\phi_2\beta}, & C_7 &= \gamma_{\tau\rho\phi_2\gamma}, & C_8 &= \gamma_{\tau\rho\phi_2\phi_3}, & C_9 &= \gamma_{\tau\rho\phi_3\beta}, & C_{10} &= \gamma_{\tau\rho\phi_3\gamma}, \\ C_{11} &= \gamma_{\tau\phi_2\beta\gamma}, & C_{12} &= \gamma_{\tau\phi_2\phi_3\beta}, & C_{13} &= \gamma_{\tau\phi_2\phi_3\gamma}, & C_{14} &= \gamma_{\tau\phi_3\beta\gamma}, & C_{15} &= \gamma_{\phi_2\phi_3\beta\gamma}. \end{aligned}$$

The expressions for the coefficients c_i are given by

$$\begin{aligned} c_1 &= b_{\tau\phi_3}, & c_2 &= b_{\tau\gamma}, & c_3 &= -b_{\tau\beta}, & c_4 &= -b_{\tau\phi_2}, & c_5 &= b_{\phi_2\phi_3}, \\ c_6 &= -b_{\phi_3\gamma}, & c_7 &= b_{\phi_3\beta}, & c_8 &= b_{\beta\gamma}, & c_9 &= b_{\phi_2\gamma}, & c_{10} &= -b_{\phi_2\beta}, \\ c_{11} &= -b_{\rho\phi_3}, & c_{12} &= -b_{\rho\gamma}, & c_{13} &= b_{\rho\beta}, & c_{14} &= b_{\rho\phi_2}, & c_{15} &= b_{\tau\rho}. \end{aligned}$$

We also have the components of the $B_{(2)}$ -form pullback,

$$\mathcal{B}_{(2)} = P[B_{(2)}] = \sum_{a,b=0}^5 b_{ab} da \wedge db, \quad (\text{E.3})$$

given by

$$\begin{aligned} b_{\tau\phi_3} &= (B_{\theta\alpha} \partial_\tau \theta - B_{\alpha\phi} \partial_\tau \phi) \partial_{\phi_3} \alpha - B_{\theta\alpha} \partial_\tau \alpha \partial_{\phi_3} \theta + B_{\alpha\phi} \partial_\tau \alpha \partial_{\phi_3} \phi, \\ b_{\tau\phi_2} &= (B_{\theta\alpha} \partial_\tau \theta - B_{\alpha\phi} \partial_\tau \phi) \partial_{\phi_2} \alpha - B_{\theta\alpha} \partial_\tau \alpha \partial_{\phi_2} \theta + B_{\alpha\phi} \partial_\tau \alpha \partial_{\phi_2} \phi, \\ b_{\tau\rho} &= -B_{\theta\alpha} \partial_\rho \theta \partial_\tau \alpha + B_{\alpha\phi} \partial_\rho \phi \partial_\tau \alpha + \partial_\rho \alpha (B_{\theta\alpha} \partial_\tau \theta - B_{\alpha\phi} \partial_\tau \phi), \\ b_{\rho\phi_2} &= (B_{\theta\alpha} \partial_\rho \theta - B_{\alpha\phi} \partial_\rho \phi) \partial_{\phi_2} \alpha - B_{\theta\alpha} \partial_\rho \alpha \partial_{\phi_2} \theta + B_{\alpha\phi} \partial_\rho \alpha \partial_{\phi_2} \phi, \end{aligned}$$

$$\begin{aligned}
b_{\rho\phi_3} &= (B_{\theta\alpha} \partial_\rho \theta - B_{\alpha\phi} \partial_\rho \phi) \partial_{\phi_3} \alpha - B_{\theta\alpha} \partial_\rho \alpha \partial_{\phi_3} \theta + B_{\alpha\phi} \partial_\rho \alpha \partial_{\phi_3} \phi, \\
b_{\phi_2\phi_3} &= (B_{\theta\alpha} \partial_{\phi_2} \theta - B_{\alpha\phi} \partial_{\phi_2} \phi) \partial_{\phi_3} \alpha - B_{\theta\alpha} \partial_{\phi_2} \alpha \partial_{\phi_3} \theta + B_{\alpha\phi} \partial_{\phi_2} \alpha \partial_{\phi_3} \phi, \\
b_{\tau\beta} &= (B_{\alpha\beta} - B_{\theta\alpha} \partial_\beta \theta + B_{\alpha\phi} \partial_\beta \phi) \partial_\tau \alpha + \partial_\beta \alpha (B_{\theta\alpha} \partial_\tau \theta - B_{\alpha\phi} \partial_\tau \phi), \\
b_{\rho\beta} &= (B_{\alpha\beta} - B_{\theta\alpha} \partial_\beta \theta + B_{\alpha\phi} \partial_\beta \phi) \partial_\rho \alpha + \partial_\beta \alpha (B_{\theta\alpha} \partial_\rho \theta - B_{\alpha\phi} \partial_\rho \phi), \\
b_{\phi_3\beta} &= (B_{\alpha\beta} - B_{\theta\alpha} \partial_\beta \theta + B_{\alpha\phi} \partial_\beta \phi) \partial_{\phi_3} \alpha + \partial_\beta \alpha (B_{\theta\alpha} \partial_{\phi_3} \theta - B_{\alpha\phi} \partial_{\phi_3} \phi), \\
b_{\phi_2\beta} &= (B_{\alpha\beta} - B_{\theta\alpha} \partial_\beta \theta + B_{\alpha\phi} \partial_\beta \phi) \partial_{\phi_2} \alpha + \partial_\beta \alpha (B_{\theta\alpha} \partial_{\phi_2} \theta - B_{\alpha\phi} \partial_{\phi_2} \phi), \\
b_{\rho\gamma} &= (B_{\alpha\gamma} - B_{\theta\alpha} \partial_\gamma \theta + B_{\alpha\phi} \partial_\gamma \phi) \partial_\rho \alpha + (B_{\theta\gamma} + B_{\theta\alpha} \partial_\gamma \alpha) \partial_\rho \theta - (B_{\gamma\phi} + B_{\alpha\phi} \partial_\gamma \alpha) \partial_\rho \phi, \\
b_{\tau\gamma} &= (B_{\alpha\gamma} - B_{\theta\alpha} \partial_\gamma \theta + B_{\alpha\phi} \partial_\gamma \phi) \partial_\tau \alpha + (B_{\theta\gamma} + B_{\theta\alpha} \partial_\gamma \alpha) \partial_\tau \theta - (B_{\gamma\phi} + B_{\alpha\phi} \partial_\gamma \alpha) \partial_\tau \phi, \\
b_{\phi_2\gamma} &= (B_{\alpha\gamma} - B_{\theta\alpha} \partial_\gamma \theta + B_{\alpha\phi} \partial_\gamma \phi) \partial_{\phi_2} \alpha + (B_{\theta\gamma} + B_{\theta\alpha} \partial_\gamma \alpha) \partial_{\phi_2} \theta - (B_{\gamma\phi} + B_{\alpha\phi} \partial_\gamma \alpha) \partial_{\phi_2} \phi, \\
b_{\phi_3\gamma} &= (B_{\alpha\gamma} - B_{\theta\alpha} \partial_\gamma \theta + B_{\alpha\phi} \partial_\gamma \phi) \partial_{\phi_3} \alpha + (B_{\theta\gamma} + B_{\theta\alpha} \partial_\gamma \alpha) \partial_{\phi_3} \theta - (B_{\gamma\phi} + B_{\alpha\phi} \partial_\gamma \alpha) \partial_{\phi_3} \phi, \\
b_{\beta\gamma} &= B_{\theta\gamma} \partial_\beta \theta - B_{\beta\gamma} - B_{\gamma\phi} \partial_\beta \phi - (B_{\alpha\beta} - B_{\theta\alpha} \partial_\beta \theta + B_{\alpha\phi} \partial_\beta \phi) \partial_\gamma \alpha + \partial_\beta \alpha (B_{\alpha\gamma} - B_{\theta\alpha} \partial_\gamma \theta + B_{\alpha\phi} \partial_\gamma \phi),
\end{aligned}$$

where the explicit form of the components of the B -field are given in eq. (A.2).

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